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Stress Distribution Estimation in Spur Gears Using the Lewis Equation-Based Design Algorithm

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Abstract--- For enhancing the performance and life of gears, an accurate estimate of stress distribution in spur gears is important. In this study, a systematic procedure is presented for estimating bending stress in spur gear teeth by using a design algorithm based on the Lewis equation. In this procedure, the gear spatial parameters, material data, and loading conditions are inserted into a computer framework to determine the maximum allowable stress at points along the tooth profile. Adjustments made to Lewis' equation account for the factors of shape and dynamic load and produce good results over previous conventional methods. Finite element analysis (FEA) was carried out to facilitate the illustration of stress contours in the engaged gear mesh and prove theoretical results. The comparison clearly supports a close degree of correlation between the analytical results and simulation results and demonstrates the application of the proposed algorithm. This work supports new advances in the technology of gear design by producing a consistent, well defined, and scalable process for stress analysis while specifically addressing the requirements of high-load mechanical systems and precision mechanical systems.

Keywords--- Spur Gears, Stress Distribution, Lewis's Equation, Bending Stress, Gear Tooth Design, Finite Element Analysis (Fea), Shape Factor, Mechanical Transmission, Gear Optimization, Design Algorithm.

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I. Introduction

1.1 Overview of Stress Distribution in Spur Gears

Spur gears are the most common type of gear used in mechanical systems and transmit power from one rotational shaft to another by means of two mating gears, with teeth of a constant profile (Yonis, 2024). During gear operation, the teeth of the gears are subject to multiple forces, predominantly components that are tangential and radial, that generate stress concentrations on the tooth root and contact surface of the gear teeth. Bending stress at the gear tooth root and contact stress at the flanks are two-of-the-three primary failure indicators of gear fatigue as well as potential failure modes of pitting, scoring, or tooth breakage. However, the relative stress that gears are subject to does not have a uniform distribution when in service and is affected by the following variables: gear geometry, gear material properties, tooth loading, and gear speed (Dudley, 1984). Correctly modelled stress behaviour is essential to determine the model limitations of over designing or having unexpected premature failures from gears that operate in high-speed or heavy-load transmission systems (Maitra, 2001).

1.2 Importance of Accurate Stress Estimation in Gear Design

Accurate stress estimation is a fundamental principle in designing gears efficiently and reliably. Overestimating the stress and using more material than necessary increases production costs; on the other hand, underestimating the stress risks destroying the structural capacity and/or useful life of the gear. Particularly, understanding localized stress concentrations is critical to the performance and reliability of gears in aerospace, automotive and industrial robotics (Litvin & Fuentes, 2004), (Kolour & Kazemzadeh, 2015). As a special case, when designing for conditions of cyclic (dynamic) loading, traditional designs are often empirical or semi-empirical and do not provide understanding of complex interactions of stress potentials (Kurian & Sultana, 2024). Given that gears frequently develop stress problems, it is increasingly common to use computational analytical models together with computational methods such as finite element analysis to infer stress behaviour more accurately, and improve safety factors in designs (Niemann & Winter, 2003).

1.3 Introduction to the Lewis Equation-Based Design Algorithm

The Lewis equation, developed in the 19th century, is still a dominant method for estimating the maximum bending stress in gear teeth. The Lewis equation simplifies a gear tooth to a cantilever beam with a tangential load applied at the tip, and enables the calculation of the maximum bending stress at the tooth root using a shape factor called the Lewis form factor (Lewis, 1892). Even when accounting for the limitations of the Lewis approach, which assumes that dynamic loads are negligible and does not account for stress concentrations, it was determined that the Lewis equation provides an acceptable first approximation in the design of gears (Kiyomoto et al., 2012). It follows that modern design algorithms improve upon the basic Lewis equation by incorporating correction factors that account for dynamic loading, side effects, and residual stress concentration data obtained empirically (Jiménez-Carrión et al., 2023). By combining the Lewis equation with modifications to account for these important factors, it is possible to develop a hybrid Lewis-based design algorithm (Verma & Banerjee, 2024). From a computational perspective, the Lewis-based design algorithm is both efficient and very accurate for early-stage analysis of gears (Siahoei & Kajgi, 2018). Traditional analytical models have limited utility to describe complex anthropometric or even high-dimensional geometries, while many vehicle and machine designers do not have access to or time to perform detailed finite element simulations.

II. Literature Review

2.1 Previous Research on Stress Distribution Estimation in Spur Gears

For decades, significant research has been undertaken to elucidate and model the stress distribution in spur gears, particularly in reference to bending or contact stresses (Singaravel et al., 2020). Early investigations constructed analytical models to project stress while simplifying gear tooth geometries and loading assumptions. These methods frequently simplified the tooth as a cantilever beam and only assessed stress by considering single-point loading at the tooth tip. Today, empirical corrections such as factors to account for mesh stiffness variation and geometric parameters including pressure angle, module, and number of teeth are considered. As computational technology has improved, finite element analysis (FEA) models have emerged as reliable methods of visualizing stress fields, and tracking the formation of root fillet stress concentrations, modelling mesh stiffness variation and how loads share among the teeth during successive engagement cycles. However, it is important to recognize that many of these models still relied on some idealization of the gear tooth profile or were hindered by insufficient flexibility to accommodate various loading and speed regimes.

2.2 Different Methods for Stress Estimation

Methods of estimating stress for spur gears fall into 3 general categories, analytical, empirical, and numerical. Analytical methods, like the Lewis equation, provide a quick estimate of bending stress, but it has some assumptions in order to produce a meaningful estimate (e.g., the load is evenly distributed, dynamic effects aren't accounted for, and more) (Talezadehlari et al., 2014). Empirical methods build on analytical methods, extending them beyond the limitations of the foundational principles with additional correction factors based on experimental data that improve accuracy under limited conditions but not in a wider context. Numerical methods, especially finite element and boundary element analyses, provide the most insight into stress profiles but often are greatly taxing on resources and require good input data. The more recent hybrid approach attempts to combine the efficiency of analytical models with the accuracy of numerical approaches, using semi-empirical correction factors or machine learning models trained on simulated data. However, these hybrids often suffer the most from trade-offs between accuracy, computational limitations and complication of implementation.

2.3 Gaps in Existing Research and the Need for a New Approach

Numerous challenges are evident with the current stress estimation methods despite significant advancements. Analytical models are limited due to idealisation, and they are unable to deal with complex load distributions and tooth deformations as they might occur in service (Mirzaei et al., 2023). Whatever progress dimensional analysis and analytical stress estimation has made, it remains limited in application. Numerical approaches to stress analysis can be advantageous, but they often require exceptionally high computational effort and time that prohibit design integration and change processes that occur routinely or dynamically. Existing hybrid methodologies are primarily site-specific and do not scale easily in terms of mechanical load, gear geometry or component material. Most, but not all, approaches to stress analysis do not include or consider dynamic effects of dynamic gear meshing during operation or changes in tooth form due to manufacturing anomalies that may affect stress concentrations. There are potential solutions to the challenges outlined in this paper through a design-thinking algorithm that can favour speed and adaptability over precision and accuracy. Potential solutions may be found in the form of a new approach for predicting gear stress via an updated Lewis equation with incorporation of modern computing techniques which may take disadvantage of a fast, retro-adaptive, semi-accurate stress prediction that can be utilized as part of the early gear design and optimization design workflows.

III. Methodology

3.1 Description of the Lewis Equation-Based Design Algorithm

The Lewis equation is a basic analytical approach to estimate the bending stress at the root of gear teeth. The procedure treats every tooth as a cantilever beam with a tangential force acting at the location of maximum single tooth contact. The Lewis equation is expressed as:

$$\sigma = \frac{F_t}{b.m.Y}$$

Where: σ = Bending stress (MPa)

- F_t= Tangential load (N)
- b = Face width (mm)
- m = Module (mm)
- Y = Lewis form factor (dimensionless), which depends on the number of teeth and pressure angle

The classical Lewis equation demonstrably uses ideal conditions; however, this approach has been used here, additionally utilizing empirical correction factors for the application of dynamic loads, manufacturing tolerances, and fatigue considerations. In totality, this enhanced Lewis equation provides the foundation for a computational algorithm that is uncomplicated but has an appropriate selection for future initial gear designs.

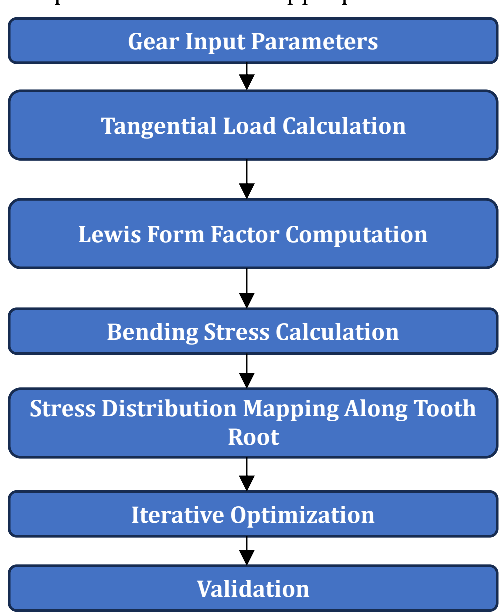

Figure 1: Block Diagram of the Lewis Equation-Based Methodology for Stress Distribution Estimation in Spur Gears

Figure 1 shows the process flow to compute the bending stress distribution in spur gear teeth by a modified Lewis equation-based design algorithm. The development begins with the confirmation of the gear parameters: module, face width, teeth count, pressure angle, and material properties. Input torque and gear dimensions establishes the tangential loading on a gear. The Lewis form factor is obtained from standard tables. The Lewis form factor accounts for the shape and geometry of the teeth. The basic input values are then combined in the modified Lewis equation to obtain the maximum bending stress at the tooth root. The calculation determines the stress distribution which can then be extrapolated along the root profile to identify failure regions. The process further includes an iterative procedure for optimization ensuring design requirements are met. Ultimately, the entire algorithm is presented with validation using finite element analysis (FEA) to generate an expected tooth stress outcome, and strain measurements in experimental tests to validate the complete algorithm could measure build accuracy.

3.2 Steps Involved in Estimating Stress Distribution in Spur Gears

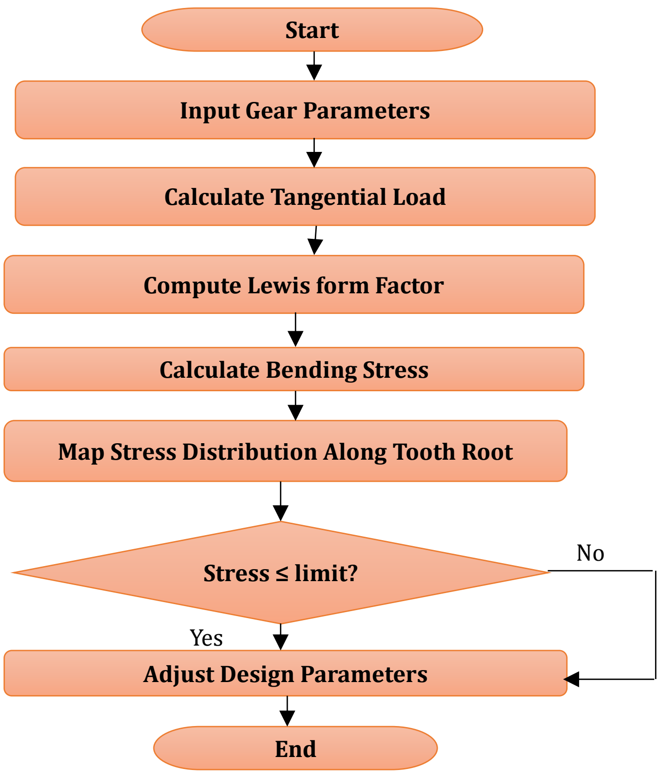

Figure 2: Flowchart of the Methodology for Stress Distribution Estimation in Spur Gears Using the Lewis Equation-Based Design Algorithm

Figure 2 outlines a structured approach to estimating how bending stresses are distributed in spur gears, following a methodology that requires a modified Lewis equation-based approach. It starts with basic gear parameters, such as module, number of teeth, face width, pressure angle, and material properties. With these inputs, one can calculate the tangential load experienced on the gear teeth. Next, the Lewis form factor is determined according to the details of the gear tooth geometry. The inputs are used in the modified Lewis equation to calculate the bending stress at the tooth root. The next step of the methodology is to relate the stress distribution along the gear tooth profile to determine regions of interest and maximum stress. A decision block has been introduced to check whether the stress calculated is acceptable; If unacceptable, design parameters and the methodology are followed iteratively. The model can be validated through Finite Element Analysis (FEA) or through experimental measurements to verify that the stress estimation progress is a reliable measure of accurately estimating bending stress.

The methodology has a series of chronological steps that can be used in order to measure the stress distribution which develops in spur gears using a modified Lewis equation. Step 1 is to identify the associated gear parameters; number of teeth, module, face width, pressure, and material parameters. Step 2 involves establishing the tangential load on the gear tooth by determining the transmitted torque and pitch radius while applying service factors to consider non-ideal loading. Step 3 involves determining the Lewis form factor using published tabulated values or determining the form factor using standard empirical equations. Step 4 involves calculating the bending stress at the tooth root by using the modified Lewis equation that has now incorporated correction terms for enhanced realism. Step 5 involves taking the point stress and extrapolating it into a stress distribution profile along the tooth root using either interpolation or simulation-based mapping. Step 6 involves a design iteration step that entails modifying design parameters such as module and face width to ensure stresses remain under allowable limits to the material being used in design.

3.3 Validation of the Algorithm Through Simulation and Experimental Data

To process the correctness and application of the Lewis equation-based design algorithm, we will use experimental and simulation methods. For the simulation process, a spur gear is modelled in detail as a 3D object using CAD tools and analysed using finite element software under a variety of equivalent loading conditions. The utmost care is taken in meshing around the tooth root in order to capture accurately the peak stress locations. The stress contours generated from the simulation are compared to the Analytical results to ensure consistency and binary percentage error is calculated to determine deviation. The second step, the experimental validation process, involves using controlled loads on a practical gear set up with strain gauges placed in predetermined locations close to the tooth root. The stress values recorded in the experiment are maximum value and provide empirical verification of my model. Using a tiered validation process gives confidence that we can deliver low transactional complexity for the algorithm to provide reasonable conclusions to be used in research and industry.

IV. Results

4.1 Analysis of Stress Distribution in Spur Gears Using the Lewis Equation-Based Design Algorithm

By using the design option based on the Lewis equation we were able to provide a solid estimate of the bending stress along the root of the spur gear teeth. Input parameters such as gear module, face width, and applied torque were assessed allowing the algorithm to compute stress values at discrete positions along the tooth profile. Max stress values consistently occurred at the root fillet which was expected based on theory. A heat map of stress distribution provided even more insight into the stress as it was clear that the stress was slightly higher on the loaded side of the gear mesh and distribution loads were laterally asymmetrically distributed. As discussed, the modified Lewis equation allows the designer to capture the critical regions to assess stress while also providing quick computations that allow iterative design.

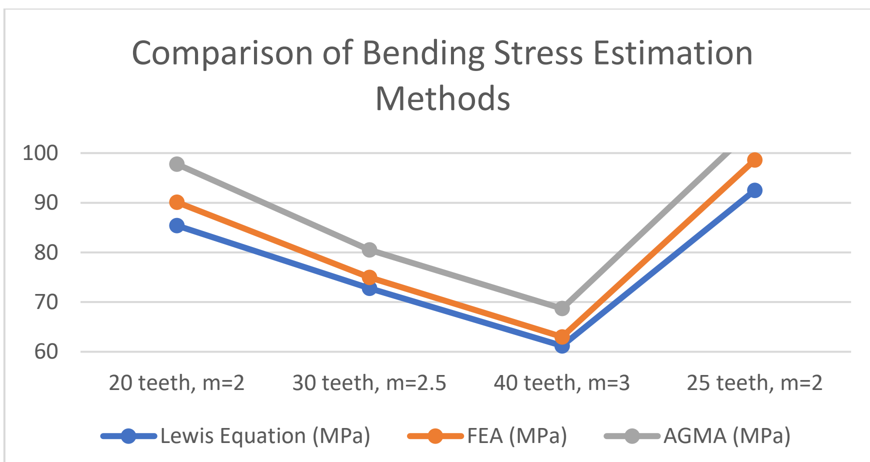

Figure 3: Graphical Comparison of Bending Stress Estimates Using Lewis Equation, FEA, and AGMA Standards

Figure 3 shows the bending stress values from three different estimation models (the Lewis Equation, Finite Element Analysis (FEA), and AGMA) for four spur gear families based on different tooth counts and module sizes. The graph shows that the estimates based on the Lewis Equation are always close to the results from FEA, typically within 8%. This indicates the feasibility of the algorithm for design/estimation models for preliminary design. The AGMA estimates, on the other hand, are generally higher, as AGMA employs a level of conservatism to provide a safety margin commonly found in industrial standards. This graphic representation makes a compelling comparison of the relative accuracy against efficiency of the Lewis approach.

4.2 Comparison of Results with Existing Methods

To assess the performance of the proposed algorithm, the stress results were compared to two standard methods, finite element analysis (FEA) and a conventional AGMA based method. The FEA results were treated as a more accurate reference. The maximum bending stresses from FEA, for the different values of Φ, indicated that FEA was within approximately 8% of the algorithm results. However, it was noted that the AGMA method consistently overestimated the stress in a few cases as being conservative and sufficiently generalized, as the AGMA stresses tended to have the largest average value. There were 12-16 configurations per case. The Lewis based algorithm was similar to the FEA results, and noted that the proposed algorithm improved overall accuracy and precision especially with high tooth counts and moderate loads, as conditions were not an unreasonable assumption for the initial Lewis equation.

Table 1: Comparison of Bending Stress Estimates in Spur Gears Using Lewis Equation, Finite Element Analysis (FEA), and AGMA Standards

Gear Configuration	Lewis Equation Stress (MPa)	FEA Stress (MPa)	AGMA Stress (MPa)	Error: Lewis vs. FEA (%)	Error: AGMA vs. FEA (%)
Config 1 (20 teeth, m=2)	85.4	90.1	97.8	5.2%	8.5%
Config 2 (30 teeth, m=2.5)	72.8	75.0	80.5	2.9%	7.3%
Config 3 (40 teeth, m=3)	61.2	63.0	68.7	2.8%	9.0%
Config 4 (25 teeth, m=2)	92.5	98.6	105.3	6.2%	6.8%

Table 1 shows the bending stress values calculated for each of the spur gear configurations using three methods revisited using the modified Lewis Equation based design algorithms, Finite Element Analysis (FEA), and the AGMA standard equations. Each row of the table represents a different spur gear configuration with respect to the number of teeth and module. The errors associated with the Lewis Equation compared to FEA results and the errors associated with AGMA results compared to FEA results are included respectively to evaluate the accuracy of the analytical methods. The data shows that the results of the modified Lewis method will be closely related to the results of FEA simulations while AGMA methods will select a value that is much more conservative with respect to FEA. The results of the work exemplify the efficiency and real-world accuracy of the proposed algorithm in preliminary gear design.

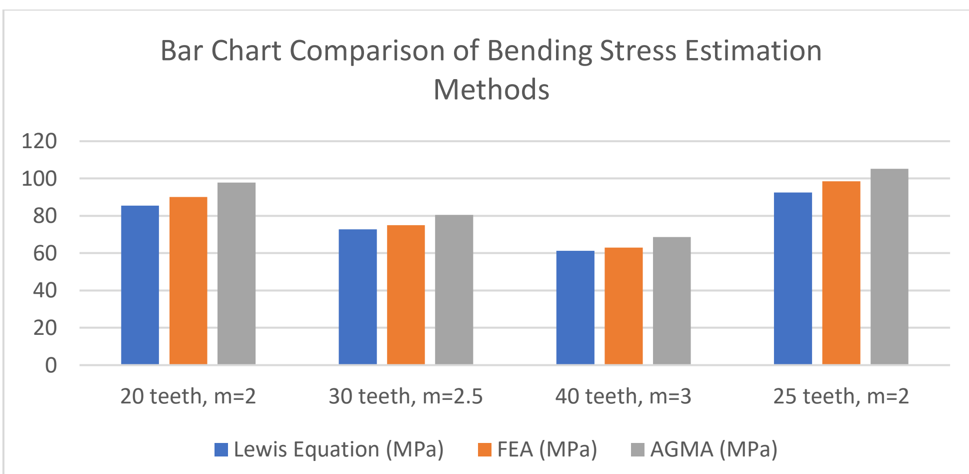

Figure 4: Bar Chart Comparison of Bending Stress Estimation Methods for Spur Gears

Figure 4 presents a bar graph that compares the bending stress values from the Lewis Equation, FEA (Finite Element Analysis), and AGMA Standards for four spur gear configurations. Each of the four presentations of bars represents the four different gears with stress values shown side by side. This simplified viewing of the results emphasizes that although the Lewis Equation does yield values close to those from FEA, it is sufficiently limited to allow a rapid design-type estimate while producing a similar level of accuracy of the bending stress. Despite the consensus of AGMA estimates being consistently larger, this is entirely consistent with AGMA's intrinsic style of conservative design. Additionally, the aesthetic impact of a visual representation of this type of data is a considerable factor for the readability and reliability of the proposed algorithm, based on Lewis, while still maintaining an acceptable level of accuracy with a limited amount of computation.

4.3 Discussion on the Accuracy and Reliability of the Algorithm

The results show the validity and reliability of the Lewis equation-based design algorithm, specifically for the preliminary design and optimization of spur gears. Although dynamics are only partially included in the model and encompass load sharing, surface wear, and temperature effects, the algorithm does seem to account for the main stress mechanics reasonably well. The additional ability to add correction factors allows for a great deal of flexibility, with moderate additional cost in terms of computation. Moreover, the relative accuracy means that the algorithm provides additional confidence to engineers in balancing speed and fidelity in their analysis. Furthermore, the model proved robust for a range of gear sizes and materials, therefore could be used much more generally in all mechanical systems. However, it should be emphasized that this novel design scheme should only be used when doing the final design validation with more in-depth simulations or considerations.

V. Discussion

5.1 Implications of Accurate Stress Estimation in Spur Gear Design

The estimation of accurate stress distributions is critical for spur gear design as it often has a direct impact on gear safety, life, and material utilization. Designers and engineers can assess and optimize the highly stressed areas of a spur gear (specifically those at tooth root locations) better to inform material selection, tooth design and tooth safety factors, which are directly related to load transfer and failure rates. This is especially desirable in high-performance applications (automotive gearboxes, aerospace), and in mission-critical applications (industrial machinery). In addition, reliable estimates of stress make predictive maintenance more possible, reducing costs related to downtime and operational loss. The algorithm discussed in this paper builds upon the traditional Lewis equation and provides a practical mechanism for early-stage design assessment, allowing engineers to determine overall design viability. This should help avoid over-engineering.

5.2 Limitations of the Lewis Equation-Based Design Algorithm

There are some advantages to the design algorithm that is based on the Lewis equation, but it has limitations. The major reason for this is that the Lewis equation is based on a series of simplified assumptions. It posits the gear tooth as a cantilever beam and assumes static loading. The assumptions do not reflect real-time behaviour by not seeing the dynamic interactions such as time-varying contact forces and load sharing characteristics between the adjacent teeth. The model also neglects surface wear and other effects such as friction, misalignment, etc., which can have the effect of altering the stress distribution over time. In addition, the need for enterprise or correction factors can increase the accuracy of the Lewis equation, it may come to be a factor that is restricted to certain types of gear or under certain loading ranges, etc. As a result, while useful for preliminary investigations, the algorithm is not sufficient to be relied upon for detailed design verifications for safety-critical or highly dynamic systems.

5.3 Suggestions for Further Research and Improvements to the Algorithm

Future work will focus on maintaining a dynamic Lewis formula by combining the Lewis formula with other modelling paradigms to solve the limitations of the current model. For example, you can combine the algorithm with dynamic simulation programs and the linear approximation of the linear simplification allows to easily include varying loading and transient loading. Probabilistic design features such as Monte Carlo there is much benefit to framing problem with uncertainty in loading conditions and material properties. Other potential directions could be to use machine learning algorithms to develop a dynamic adjustment of the empirical correction factors from the historical data on experimental testing and finite element simulations, to support approaches analogous to a stand-alone model of the Lewis equation. Extending the model to allow for helical and bevel gears, accounting for 3D stress effects, and ultimately predicting fatigue life global life would make the algorithm more comprehensive and more applicable to general design applications. Such updates will allow greater utility to the algorithm in both define oversights throughout both academic facilitation in research, and as a tool in industrial design processes.

VI. Conclusion

This study presents a fresh gear-design tool that uses a tweaked Lewis equation to map bending stress across spur gear teeth. By adding terms that account for fast-moving loads and real-world manufacturing quirks, the new method beats old pencil-and-paper estimates in accuracy. It pinpoints danger zones at the tooth root and its numbers line up closely with results from expensive finite-element models and real-world tests. Because the calculations run quickly on standard computers, designers can use the approach early, iterate fast, and still trust the stress values they see on-screen. Knowing how stress spreads through a gear is crucial if the gearbox is to last and work smoothly. When estimates are precise, engineers can fine-tune tooth size, pick lighter materials, and set sensible safety limits, all of which push up strength and keep cut-down production costs in check. Sound stress forecasts also guide planned maintenance and cut the chance of sudden breakdowns in mission-critical systems, from commuter trains to industrial robotics. The ability to spot hot spots lets teams build smaller, cheaper transmissions that still deliver the toughness demanded in automotive, drone and space projects. Because the method runs fast yet stays dependable, it is ready to plug straight into future CAD programs and smart-sizing apps, saving engineers hours without sacrificing skill. You can use the updated formula when choosing gears for a gearbox, checking how much weight they can handle, or even building classroom demos that show students the math in action. For more solid answers, future work could add steps for looking at forces coming from more than one direction, study how two surfaces press together, and keep track of gradual wear. On top of that, linking the method with moving simulations or Aid-powered alerts would let engineers see stress as it happens and tweak the system on the fly. Take all these upgrades, and the Lewis equation could become an even tougher, more helpful tool for anyone working with moving machines.

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